Passport to the Cosmos

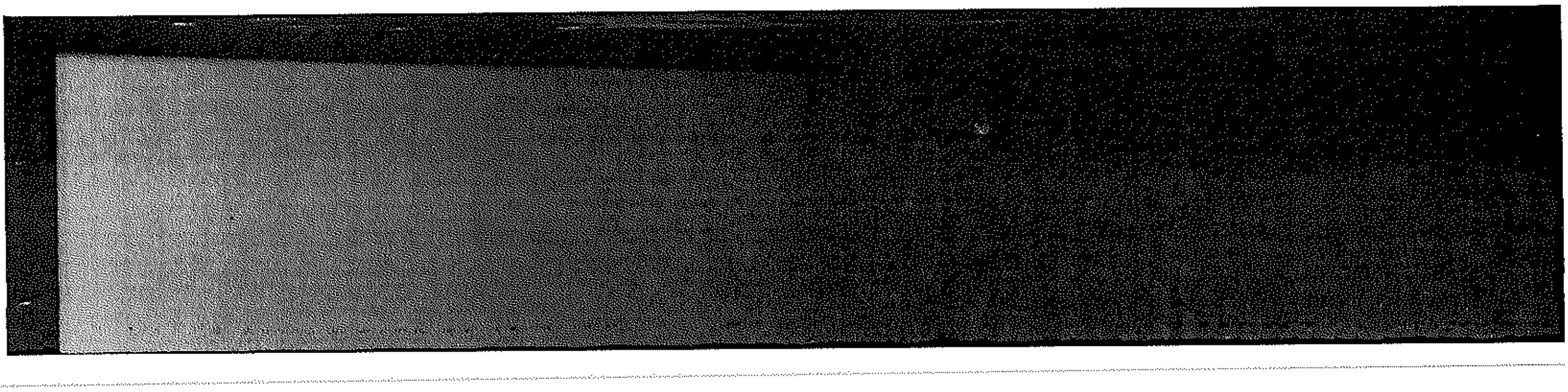
Marcus du Sautoy

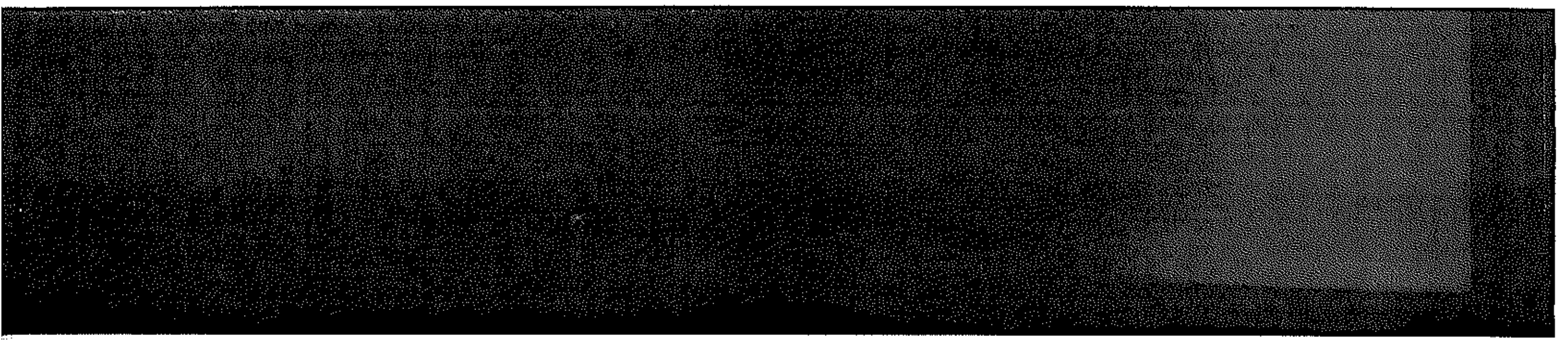
For 2,000 years, geometry was flat. Ever since Euclid published his mathematical treatise *The Elements* c. 300 BC, the angles of triangles had added up to 180 degrees. Parallel lines did not meet. Then something dramatic happened. In the first part of the nineteenth century, three mathematicians discovered that Euclid's geometry was not the only one out there. In one of the most exciting and revolutionary moments in the history of mathematics, these thinkers took us into new worlds where the certainties of Euclid were turned upside down.

Gazing at Anish Kapoor's mirrored sculptures, I find that it is into these strange geometries that the artist is taking me. The space around us that we navigate in our everyday lives feels flat and Euclidean, but look into *Sky Mirror*, 2006, or *Non-Object (Spire)*, 2007, and the geometry of our environment looks very different. Despite the virtual world that these mirrors create, they are in fact a better reflection of the reality of the space we inhabit. The scale of the planet's curvature compared to our human dimensions gives us the impression of a flat earth, but our small scale denies us the chance to see the curved surface on which we live. Travel beyond the confines of our planet into the depths of the universe and the geometry of space is even stranger than we can ever imagine. Kapoor's curved mirrors provide a lens that shows the universe as it really is, where light is warped on its way through space and our intuition is turned inside out.

Reflections

Kapoor's sculptures have a perfection and symmetry that are immediately appealing to a mathematician. *C-Curve*, 2007, or the *Sky Mirrors* are first of all extraordinary feats of engineering. Any bump or dent would spoil the illusion. The metal has to be smoothed away until no evidence of its construction is left. What remains is a perfect piece of geometry. The engineer has vanished; the mathematician remains. Each mirror is a physical realisation of a shape whose natural home is in the mind's eye of the mathematician. The sculptures are particularly striking because Kapoor does not site them in the clean abstract space of a gallery, but in





the messy physical space of the natural landscape or the urban jungle. The contrast between the abstract pure world of mathematics and the chaotic world we inhabit is stark. Yet the mastery of these pieces is the way in which the mirrored surfaces fuse these two worlds.

Curved mirrors, of course, are not a modern invention. Legend has it that in 212 BC Archimedes repelled the Roman fleet, intent on laying siege to the island of Syracuse, by using mirrors to focus the sun's rays. Although this weapon was made up of many flat mirrors, their combined effect was to act like a huge concave mirror, concentrating the sun's rays to burn the Roman fleet. In the eleventh century, the Islamic scientist Alhazen described why such concave mirrors acted as lenses. But it was not until 1668 that Isaac Newton realised that using mirrors rather than lenses in a telescope could solve a problem called chromatic aberration. As his discoveries on optics revealed, a prism separates light into its constituent frequencies – great for understanding light, but bad news if you are trying to look at distant stars. By using curved mirrors in telescopes rather than lenses, the integrity of the light could be maintained rather than being defracted. Although such mirrors have a long scientific heritage, Kapoor is the first, I believe, to exploit them so dramatically as pieces of art.

One of the wonderful things about these curved mirrors is that they instantly invite you to move, to explore the ways in which the image changes as you negotiate the space in front of the sculpture. Kapoor's mirrors tap into the same effect that Baroque architects achieved in their buildings. Instead of static squares and rectangles, the parabolas and ellipses that Bernini and Borromini used to build the churches of Sant'Andrea al Quirinale and San Carlo alle Quattro Fontane in Rome demand that you move inside the space to experience their geometry. Without moving, you don't get it. Theirs is the architecture of theatre and illusion. Kapoor's mirrors have a similar Baroque character. You don't get them until you move. And when you do, strange things happen to your world.

But it is the mathematics that Kapoor is tapping into with these mirrors that makes them so magical. There is an important distance from the mirror called its focal length. This is the point at which the parallel beams of light from the sun are focused. If you stand further away from the mirror than this focal length, then your image is inverted.

As you move towards the mirror, your image grows until you hit the focal length, when suddenly it explodes, becoming infinitely large.

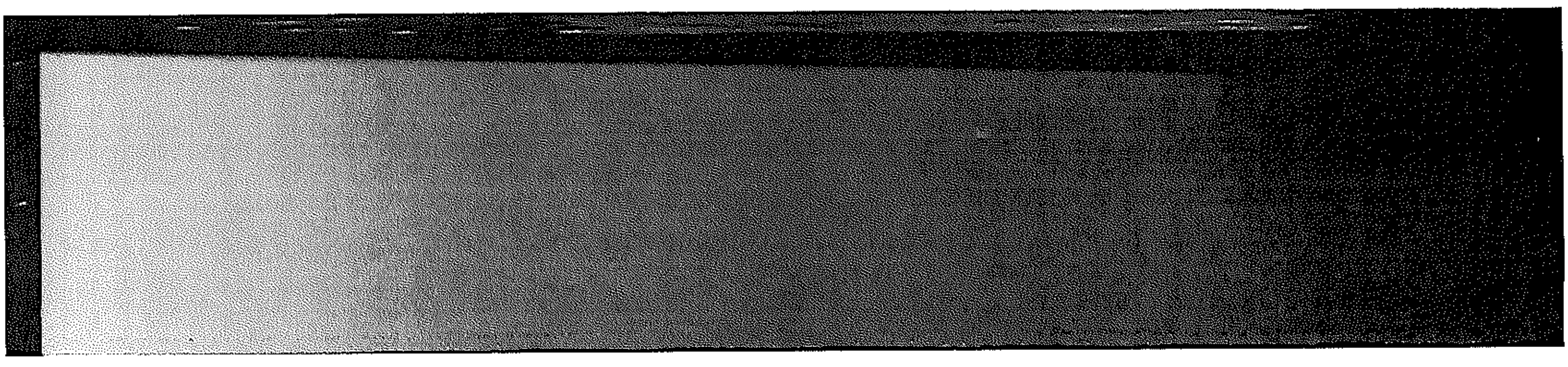
Capturing the infinite in a finite mirror is something that is particularly appealing to Kapoor. He has talked previously about how his early work with pigment allowed him to sculpt the void. Those pieces almost seem to draw the light in like a black hole, creating a sense of nothingness. Kapoor's mirrors turn that inside out. Here the light is being used to capture infinity. It is not only the infinite image that you get at the focal length that allows one to see infinity. The multiple copies produced by the mirror reflecting itself also provide the viewer with a glimpse of the infinite. Zero and infinity are two mathematical ideas that are constant themes in Kapoor's work. Perhaps it is not surprising to discover that these mathematical ideas had their origins in Kapoor's birthplace of India.

As you pass the focal length and get up close to the mirror, suddenly your image turns the right way up. The world is normal again. The mirror is acting more like the mirror you are used to. One intriguing difference between what happens either side of the focal length is that near the mirror your image appears to be on the other side of the surface, like a conventional mirror. It is a virtual image. But move back to the point at which your image is turned upside down and the light appears to emerge from an image sitting in front of the mirror. It is what is called a real image. Place a screen at this point and you can project the image onto the screen.

These real images can be exploited to create a bizarre optical illusion. Place two concave mirrors on top of each other, one with a hole in the centre, then put an object inside the chamber you've created. A copy of the object seems to float in the space where the hole is located. It feels so real that you think you should be able to pick it up. Some of Kapoor's concave mirrors play with the disconcerting effect at the heart of this illusion so that it becomes difficult to see where reality ends and the virtual begins.

Telescopes of the Mind

The strange and playful effect of these mirrors as we gaze at our place in our surroundings is why they are so immediately appealing. But for me, there is another much more subtle



and important layer to Kapoor's sculptures, which talks to our place in the cosmos. The world that you see inside these mirrors is built from new geometries that were only revealed at the beginning of the nineteenth century. In classical Euclidean flat geometry, if you draw a straight line and take a point off that line then there is only one line through that point that is parallel to the first line. The lines run like railway tracks through the geometry. One consequence of this axiom is that the angles in a triangle add up to 180 degrees. But for many centuries mathematicians had been questioning whether this was true in all geometries. Then, almost simultaneously and independently, three mathematicians, Carl Friedrich Gauss in Gottingen, János Bolyai in Transylvania and Nikolai Lobachevsky in Kazan, produced new geometries in which parallel lines and triangles behaved very differently from Euclid's expectations.

Geometrical Model of Hyperbolic Space
Bolyai Museum, Târgu Mureş

Bolyai was only twenty when he made his discovery while posted with his army unit in Temesvár. You can hear the young mathematician's excitement in the letter he wrote back to his father about this new door he'd opened on the world of geometry:

> I have found beautiful things, that surprised even me, and it would be a pity to lose them; my Dearest Father will see and know; I cannot say more, only that from nothing I have created a strange new world.[1]

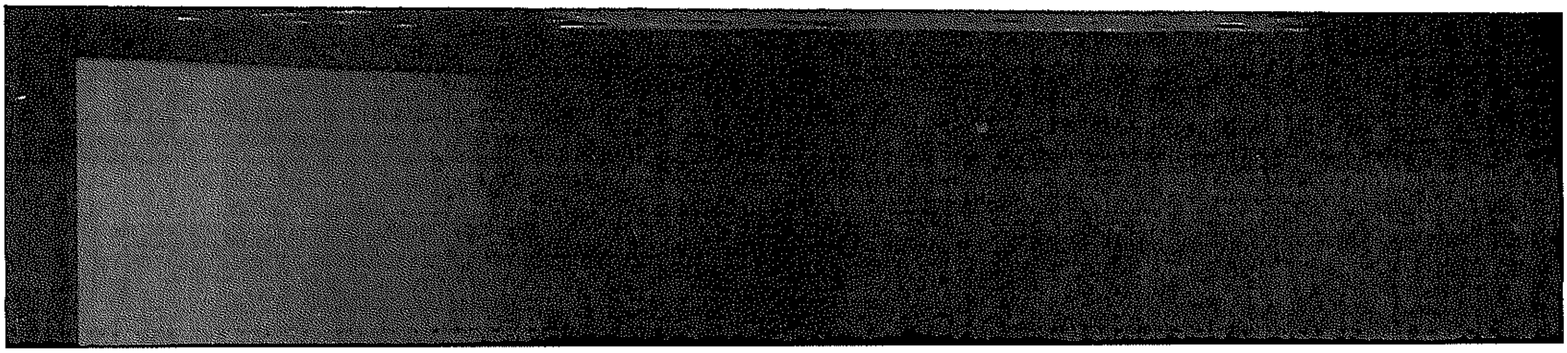
On a recent visit to the Bolyai museum in Târgu Mureş in deepest Transylvania, I discovered a model that brings to life this strange new geometry. It is the same shape as the one Kapoor has captured in his intriguing sculpture *Non-Object (Spire)*, 2007.

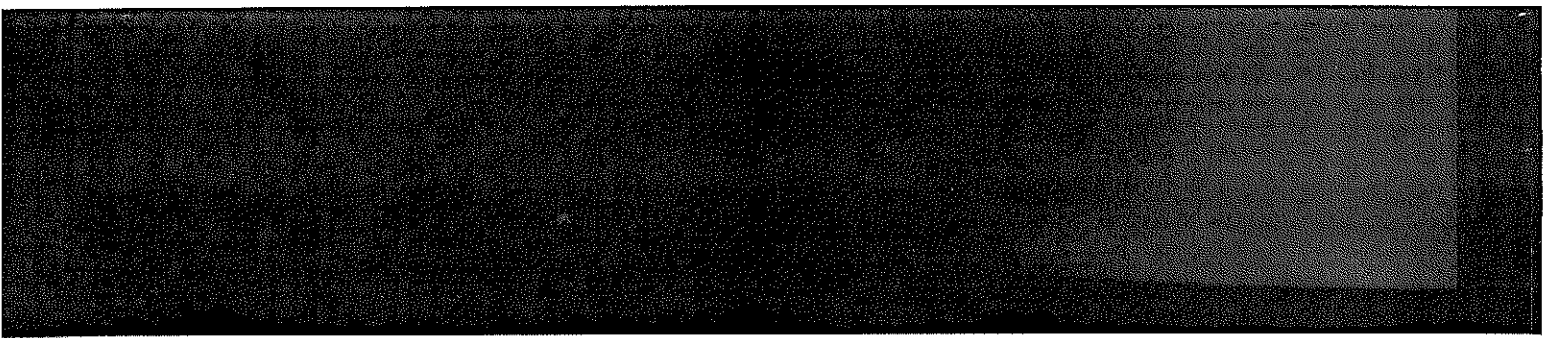
If you draw two points on the surface of *Non-Object (Spire)* and connect them with a 'straight line' (which is the shortest distance between these two points), the resulting line is not straight, but bends. Place three points on the surface and construct the triangle between these points and you get a shape whose angles add up to less than 180 degrees. Hold up a Euclidean triangle to the mirrored surface and it gets mapped to one of these non-Euclidean triangles by the mirror. *Non-Object (Spire)* is an example of this new geometry discovered by Bolyai called 'hyperbolic geometry'.

In contrast to *Non-Object (Spire)*, the concave *Sky Mirror*, 2007, produces a geometry that bulges the other way. Draw three points on the concave mirror and join them up and you get a triangle whose angles add up to more than 180 degrees. In fact, we live on the surface of such a geometry. Take a flight from London to San Francisco and the shortest path is not the straight line you'd expect, but a path that curves over Greenland and Canada. Draw a triangle connecting the pole with two points on the equator and the angles add up to more than 180 degrees.

These surfaces are two-dimensional sheets that are curved in our three-dimensional universe to give strange new geometries. The mathematicians of the nineteenth century believed that our three-dimensional universe was equally curved. They began to contemplate the bizarre idea that the straight line followed by a path of light would bend in space in a similar manner to the way in which the curvature of the

1
Letter from János Bolyai to his father
3 November 1873
See: http://www-groups.dcs.st-and.ac.uk/~history/Extras/Bolyai_letter.html





earth forces the flight path of an airplane to curve. Gauss believed this bending of light might even be observable.

While surveying the mountains in Hanover, Gauss used some of his measurements from the hill tops around Gottingen to test whether the triangle created by beams of light running between the hill tops might not actually contradict Euclid's belief about the sum of the angles in a triangle adding up to 180 degrees. We realise now that Gauss was working on too small a scale to observe any significant bending of space to counter the view of a Euclidean world. But at the beginning of the twentieth century, observations of light from stars now support his initial hunch.

It was the discovery of these geometries that led ultimately to one of the most significant discoveries in science: Einstein's theory of relativity. If you try to measure the distance between two points in space time using Euclid's geometry, all sorts of worrying paradoxes emerge. But as soon as Einstein used the strange curved geometries of Gauss, Bolyai and Lobachevsky, the paradoxes dissolved. It was on the shoulders of these revolutionary geometers that Einstein stood when he revealed that the universe is much stranger than we could ever imagine.

As you stare into Kapoor's mirrors, it is these geometries that you are staring at. For me, that is what is so exciting about these sculptures. Kapoor is like a modern-day Newton, using not telescopes but works of art to give the viewer a glimpse into the depths of the universe. These mirrors allow us to view the strange bending of space that only reveals itself on a cosmic scale in the comfort of Kensington Gardens. They may seem to turn the world upside down, but that is what it really looks like out there.