and do not know how the drawing will look like in the end. I follow the dots and lines. I don’t make a sketch before. Although one looks at the final picture of the complete network, there is a strong sense of time captured inside these drawings. They start with one point and gradually the lines unfold. Although the final picture hides the journey, it still begs the viewer to reconstruct the narrative of creation.

Often some guide is offered in the form of letters attached to the dots. The letters from the words ‘Ana’ or ‘Anta’ often try to guide you through the maze. Sometimes it is a binary code that seems to permeate the network. Interesting games emerge in each network. Is it possible to write the names so that they can be read in multiple ways? Or do you get boxed in where a dot wants an ‘X’ from one direction and an ‘N’ from the other.

How many different ways are there to go from one corner of the network to an opposite corner? It is a question that used to fascinate me when I lived in a town in Guatemala constructed on a system of avenidas and calles. We lived at one corner of the town. The supermarket was at the other. We used to play a game to choose a different route home each time we went shopping. Hefuna’s lattices beg the same question.

One can also navigate the drawings as a set of interconnected rooms. In fact my notebooks have been full of such networks of rooms with journeys through them. These sketches were stimulated by my reading of Borges’s ‘The Library of Babel’. In the story a lattice of hexagonal rooms is connected together like a beehive. The interest for the mathematician is that each room has two doors. Once you enter one door there is no choice but to leave by the other door. So is it possible to make a journey through the library visiting every room?

In Hefuna’s ‘library’ the rooms are mostly conventional squares. There is a Cartesian feeling to the construction of these chambers. But here for me another interesting mathematical characteristic of these images emerges which relates to one of the most important mathematical innovators of the 19th century.

**Riemannian Manifolds**

Bernhard Riemann introduced a whole new mathematics of navigating space and geometry. He articulated his new geometry in his habilitation lecture: ‘Über die Hypothesen, welche der Geometrie zu Grunde liegen’ (On the hypothesis on which geometry is based). So revolutionary would be his vision that it would ultimately be the mathematics that Einstein and Minkowski would need in place to describe the theory of relativity.

One of the key ideas he introduced was that of describing structures locally, within a small region, by classical Euclidean geometry. These local patches of the space can be mapped by a Cartesian grid. Yet these local pieces could be pieced together to make a global object that had a complicated geometry. Many of the theorems of Riemannian geometry are about deducing global structure from local information. A metaphor for Art? For example, can you tell how many holes the global structure has from local information about the shape?

Hefuna’s lattices are often built up of squares. Locally they are little Cartesian two dimensional worlds. But then as they evolve and connect together, their global structure can take on a variety of different structures. Hefuna is building different manifold structures as these lattices emerge. Sometimes they maintain quite a rigid two-dimensional lattice throughout maintaining the structure of a flat Euclidean
Example of Riemannian manifold whose local structure is made from Cartesian lattices.

A sense of the shape running endlessly to infinity is created. Some of Hefuna’s manifolds wrap round and create a finite geometry.

Other times, the lattice starts to collapse in on themselves. Sometimes this is achieved by squares collapsing to triangles. But more interesting is when the squares remain and the way the squares are pieced together collapses the shape. Some shapes cannot be globally mapped by squares alone. A sphere ultimately must have a singularity at the poles. Here we see triangles appearing. But it is still possible to break the sphere up into pieces that individually can be described by lattices of squares. It is this feature which leads to our impression of the world being flat. At a human level we experience the local patching. We map out our cities and our countries using two-dimensional grids. But globally there isn’t such a mapping. It is of course the reason that we have had such trouble making maps of the earth. A torus or bagel shape in contrast can be globally mapped with squares. It is what we call a flat geometry. It is one reason that slices of tori are often used by architects in the construction of buildings with
interesting curves. The Sage at Gateshead for example is made up of 27 slices of a torus. It can be constructed out of flat pieces yet when pieced together one ends up constructing a global object that has this beautiful curvaceous structure, like a wave running along the Tyne.

Hefuna's drawings are like an explorer discovering that her local world is flat. But as she travels further through her world, so different shapes emerge capturing her global universe just in the same way that cartographers and astronomers have been doing for millennia.

The connection between Hefuna's drawing and the mathematics of manifolds is further accentuated by the layering of the drawings. There is a sense of the geometry changing and evolving as one goes from the two dimensional layers to the three dimensional structure. These aspects of the geometry behind Hefuna's networks have a distinctly German heritage. But to complement this Germanic side of Hefuna's work there is also a strong Arabic theme running beneath these structures: symmetry.

Symmetry

Hefuna's lattices resonate strongly with her interest in the mashrabiya screens of Cairo. These screens are full of one of the principle themes one sees running through Arabic art. The use of symmetry to produce interesting ways of covering two-dimensional walls, floors or ceilings is found across the Muslim world, from the Moorish city of Alhambra in southern Spain to the Iranian city of Isfahan.

Although the walls offer the artist with yet another canvas to express their art, the writings of the Muslim hadith, which interpret the Koran, have put some limitations on the artist. It is forbidden under Muslim law to depict the image of any living creature. So instead the artist has been forced to express the majesty of creation through more geometric games. And that is what makes palaces like the Alhambra such a feast for a mathematician making a journey through the Arabic world.

But of course it isn't just the Arabic world that is drawn to symmetry. Living organisms are evolutionary programmed to pick out things with symmetry, to recognize a message contained inside structures with symmetry. Those that can spot a pattern with reflexive symmetry in the chaotic tangle of the jungle are more likely to survive. Symmetry in the undergrowth is either someone about to eat you or something you could eat. Our brains seem hard-wired to find meaning in symmetry. This desire to find meaning in things with symmetry is key to the use of symmetry in unlocking the state of the human mind.

This is why Hermann Rorschach developed in the 1920s his ten symmetrical inkblots as a means of unlocking a patient's unconscious mind. He believed that humans are so compelled to find a meaning or a message when shown something symmetrical that the patient's response can reveal clues to their psychological state of mind. Carl Jung also thought that symmetry was important in understanding the unconscious. Rather than the butterfly like ink-blots of Rorschach, Jung was drawn to the symbolism contained in the Hindu or Buddhist mandalas.

For Jung the mandala was an expression of the self: 'I sketched every morning in the notebook a small circular drawing, a mandala, which seemed to correspond to my inner situation at the time. With the hope of these drawings I could observe my psychic transformations from day to day.'

Jung also used his patient's drawings of mandalas as a doorway into their subconscious world. He believed that the act of creating these symmetrical images was also therapeutic in its own right helping the patient to express the different facets of their personalities.
'Most mandalas have an intuitive, irrational character and, through their symbolical content, exert a retroactive influence on the unconscious. They therefore possess a 'magical' significance, like icons, whose possible efficacy was never consciously felt by the patient.'

Given the power of symmetry to explore one's state of mind I was very struck when I talked to Hefuna about the process of making her lattice drawings. For her they are very much used as a reflective process.

'I need a kind of 'empty brain', a time when I can be just 'here and now' in the moment to be able to do drawings. For me it's like meditation. I start in the morning (that's the time when I can work best) and work all day when I do the drawing series. During such a period of time I don't do anything else for days, weeks. Only drawing. I don't pay attention to any public obligations for that time, have nearly no contact to the outside world, I totally concentrate on drawing.'

It is as if these lattices are an expression of the complex network of the mind itself. After all the brain at one level is simply a huge interconnected lattice of 100 billion neurons, linked with up to 10,000 synaptic connections each. Each drawing is like a little window on Hefuna's state of mind.

I went away from that evening with my mind buzzing with the mathematical themes bubbling beneath Hefuna's work. She very kindly gave me one of her publications full of reproductions of these lattices. On the train back home I spent my time criss-crossing the lattices, following paths, hitting dead-ends. Hefuna's work for me is all about journeys of the mind – a little like doing mathematics.

Marcus du Sautoy
Simonyi Professor for the Public Understanding of Science
and Professor of Mathematics at the University of Oxford
2009